

1.1 Digits and Place Value and Introduction to Decimals

1. Understand Digits and Place Value with Decimals

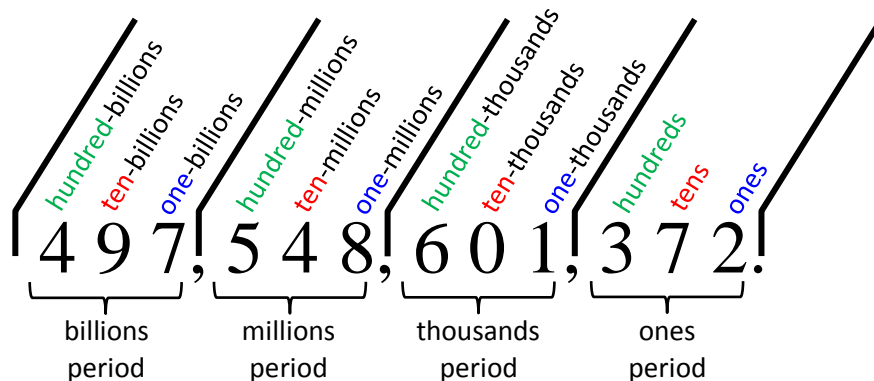
Digits are mathematical symbols that are arranged in a specific order to represent numeric values. There are ten different digits in our number system. They are listed below.

0 1 2 3 4 5 6 7 8 9

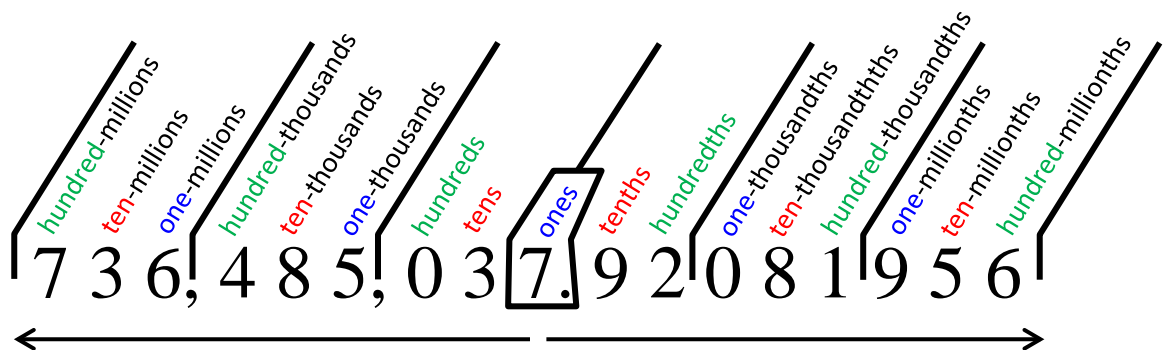
We use these ten digits (or ten symbols) to create numbers by placing them in a specific order. It is the position of each digit within a number that determines its place value. One digit alone can also represent a number. A single digit that represents a number is said to be in the *ones* place value position.

To assist us in determining place value, we use commas to separate *periods* of a number, and also use a decimal point to define the location of the *ones* place. The *ones* place is just to the left of the decimal point. When writing down whole numbers we normally do not write down the decimal point. In this case it is understood that the digit furthest to the right, or rightmost place, is in the *ones* place.

We will now look at a whole number with four full periods. The name of each period as well as the place value of each digit is labeled. Can you see a pattern in the diagram below?



Next we have a number that has digits to the right of the decimal point. Be sure to again look for a pattern by imagining the *ones* place as the middle of the number.



Can you see the pattern that is mirrored about the *ones* place? Once we learn how to identify the place value of digits, we then can learn how to read and write numbers properly.

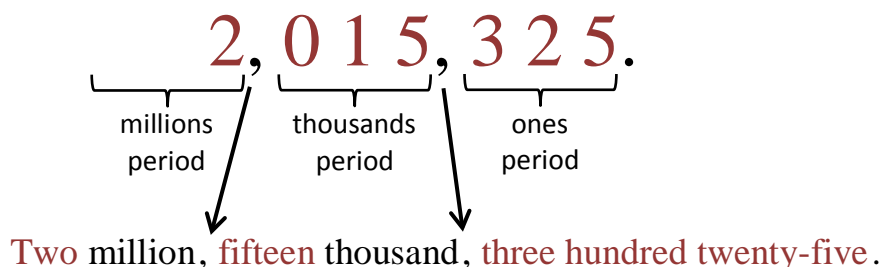
Example 1: Write down the place value of the digit 4 in the following numbers. Use the place value diagrams on the previous page to help find the answer.

- | | |
|------------------|---|
| a) 114,235 | The four is in the one -thousands place. |
| b) 2,297,465 | The four is in the hundreds place. |
| c) 0.0004 | The four is in the ten -thousandths place. |
| d) 10.259843 | The four is in the hundred -thousandths place. |
| e) 0.1030804 | The four is in the ten -millionths place. |
| f) 4,250,006,258 | The four is in the one -billions place. |

2. Review How to Read and Write Whole Numbers

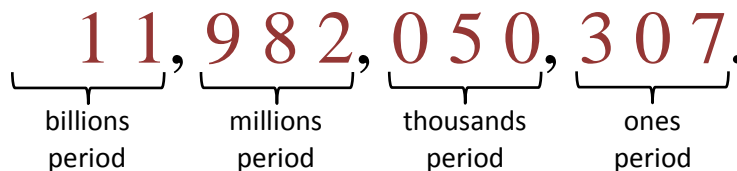
Knowing place values as well as knowing how the periods of a number are ordered, enables us to read and write whole numbers correctly.

When writing whole numbers using words, we always include the period(s) in our word statement with exception of the *ones* period. The diagram below will help us write the number 2,015,325 using words. Pay close attention to the numbers within each period and how the commas are used in the word statement below.



In the sentence above, notice how the commas break up the sentence to define the periods. Note that the *ones* period is excluded. Also, notice that we do not use the word “and” when writing down whole numbers using words. The word “and” is used to connect the decimal (or fractional) parts to the whole number. This will be addressed later in this section.

Now let’s write the number 11,982,050,307 using words.



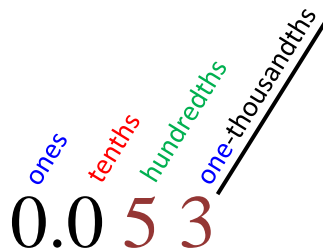
Eleven billion, nine hundred eighty-two million, fifty thousand, three hundred seven.

Once again, notice how the commas break up the sentence to define the periods. Also, notice that the *ones* period is again excluded from the word statement.

3. Understand How to Read and Write Decimal Numbers Less Than 1.

Next we will learn how to correctly read and write decimal numbers less than 1. Let's begin with 0.053 which represents a number less than 1.

To write a decimal number less than 1 using words, we first need to define the place value of the digit furthest to the right. In the number 0.053, the digit 3 is in the rightmost place. Using our place value pattern, we can see that the digit 3 is in the one-thousandths place.



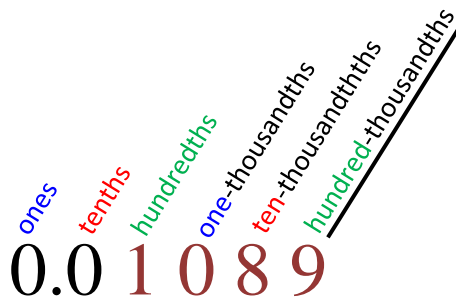
Next, we write down the number to the right of the decimal point. In this case we have the number 53. Because the 53 terminates in the one-thousandths place, it means we have “fifty-three one-thousandths”. So we write this number using words as follows.

Fifty-three one-thousandths.

Recall that a decimal number represents a fraction. Therefore we can express 0.053 as $\frac{53}{1,000}$ and both are written using words as “fifty-three one-thousandths”. Notice that the numerator of the fraction $\frac{53}{1,000}$ is represented by the numeric value to the right of the decimal point.

Note: In many cases it is acceptable to write down “fifty-three thousandths” rather than “fifty-three one-thousandths”. Check with your instructor to see if this is acceptable.

Now let's try the number 0.01089 which is again a number less than 1.



In this case, the number to the right of the decimal point is 1089 and it terminates in the hundred-thousandths place. Notice that the digit 9 is in the rightmost place. This means we have “one thousand eighty-nine hundred-thousandths”.

Therefore we can express 0.01089 as $\frac{1,089}{100,000}$ and write the number using words as follows.

One thousand eighty-nine hundred-thousandths.

Example 2: Write each of the following numbers using words. Also represent each decimal as a fraction for parts e) – h).

- | | |
|--------------------|---------------|
| a) 52,003 | e) 0.9 |
| b) 907,000 | f) 0.085 |
| c) 84,000,250 | g) 0.0030 |
| d) 108,581,609,004 | h) 0.00000406 |

Notice in parts a) – d), the numbers given are whole numbers.

Remember, when writing whole numbers using words, we always include the period(s) in our word statement with exception of the *ones* period.

- a) **52,003**. Notice we have 52 in the thousands period, and 3 in the ones period.
Fifty-two thousand, three.
- b) **907,000**. Notice we have 907 in the thousands period.
Nine hundred seven thousand.
- c) **84,000,250**. Here we have 84 in the millions period, and 250 in the ones period.
Eighty-four million, two hundred fifty.
- d) **108,581,609,004**. Here we have 108 in the billions period, 581 in the millions period, 609 in the thousands period, and 4 in the ones period.
One hundred eight billion, five hundred eighty-one million, six hundred nine thousand, four.

Notice in parts e) – h), the numbers are less than 1.

To write a decimal number less than 1 using words, we first write down the numeric value to the right of the decimal point, followed by the place value of the rightmost digit.

- e) **0.9** Here we have the number 9 to the right of the decimal point and it is in tenths place.
Nine tenths.
- f) **0.085** Here we have the number 85 to the right of the decimal point. The 5 is the rightmost digit and it is in the one-thousandths place.
Eighty-five one-thousandths.
- g) **0.0030** Here we have the number 30 to the right of the decimal point. The 0 is the rightmost digit and it is in the ten-thousandths place.
Thirty ten-thousandths.
- h) **0.00000406** Here we have the number 406 to the right of the decimal point. The 6 is the rightmost digit and it is in the hundred-millionths place.
Four hundred six hundred-millionths.

Answer the following questions.

1) Write down the place value of the digit 7 in the following numbers.

- a) 947,025 e) 0.007000
 b) 306.007 f) 0.065070
 c) 580.85670 g) 9.871324
 d) 657,289,634 h) 6.0578238

2) Using the number below, identify the digit in the given place value.

20,546,318.72968467

- a) one-millions
 b) ten-millionths
 c) one-thousandths
 d) hundred-thousands
 e) hundreds
 f) hundredths
 g) one-millionths
 h) ten-thousands

3) Write each of the following numbers using words.

- a) 500,009
 b) 0.0018
 c) 456,800
 d) 0.00507
 e) 13,000,060,105
 f) 0.08060

4) Write each of the following numbers using digits.

- a) Seventy-five one-thousandths.
 b) One hundred eight million.
 c) Sixteen ten-millionths.
 d) Thirty-three thousand.
 e) Four million, six-hundred seventy-five.
 f) Ninety million, two thousand, one hundred four.

4. Understand How to Read and Write Numbers

The number 125.87 has a whole number part and a decimal (or fractional) part. The whole number part represents a quantity that is greater than 1, and the decimal part represents a quantity that is less than 1.

The whole part of the number 125.87 is 125 and is read “one hundred twenty-five”. The decimal part of the number is .87 and is read “eighty-seven hundredths”. The decimal point is used to connect the whole number part to the decimal (or fractional) part by addition. This means that the number 125.87 actually represents a mixed number!

$$125.87 = 125 + .87 = 125 + \frac{87}{100} = 125\frac{87}{100}$$

Recall that the mixed number format represents a sum of a whole number part and a fractional part.

To write the number 125.87 using words, we first write down the whole number part. Next, we use the word “and” to connect the whole number part to the decimal (or fractional) part.

125.87



One hundred twenty-five and eighty-seven hundredths.

Algebra2go: Review of Place Value and Introduction to Decimals

Suppose we are given the number 1,002.0050 which again has both a whole number part and a decimal (or fractional) part. The whole number part is 1,002 and is written “one thousand, two”. The decimal part of the number is .0050 and is written “fifty ten-thousandths”.

As before, to write the number 1,002.0050 using words, we first write down the whole number part. Next, we use the word “and” to connect the decimal (or fractional) part.

$1,002.0050$
 ↓
 One thousand, two and fifty ten-thousandths.

When we need to write out a check, we must always indicate the dollar amount in two forms. First we write the number using digits, and second we write the number using words.

Example 3: In the appropriate space, write in the dollar amount of the check using words.

Patrick Quigley	0352
1234 Main St	Date: <u>7/30/12</u>
Everywhere, CA 01987	
PAY TO THE ORDER OF <u>Summer Rentals</u> \$ <u>1,834.18</u>	
DOLLARS	
MEMO <u>Vacation</u>	<u>Patrick Quigley</u>

The dollar amount of the check is 1,834.18 which has both a whole number part and a decimal (or fractional) part. To fill in the indicated dollar amount using words, we write the following words on the dollar amount line in the check above.

$1,834.18$
 ↓
 One thousand, eight hundred thirty-four and eighteen hundredths

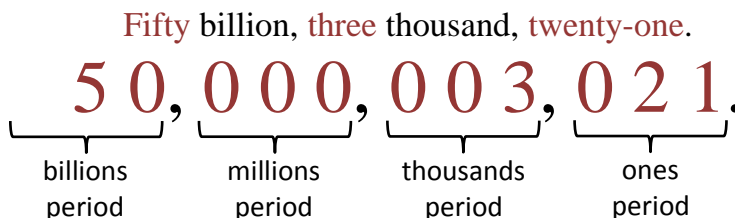
It is also acceptable to write the decimal part as a fraction.

$1,834.18$
 ↓
 One thousand, eight hundred thirty-four and $\frac{18}{100}$

Now we will look at how to write a number using digits given a word statement. We will begin with a whole number. The word statement we will work with is written below.

Fifty billion, three thousand, twenty-one.

Notice that a *millions* period is not present in the word statement above. When writing the number using digits, the *millions* period must be included. To represent the *millions* period in this case, we place three 0's within this period. The result is represented in the diagram below.



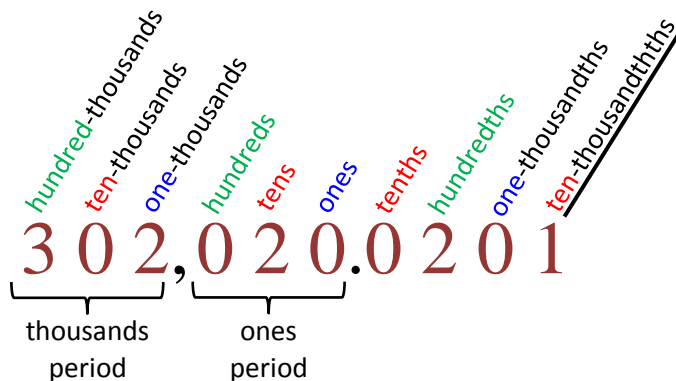
Observe the three 0's in the *millions* period. These zeros are required in order to represent the number “fifty billion, three thousand, twenty one” correctly.

Additionally, notice that there are always three digits between any two commas. In the case of the *ones* period, always remember that it must contain three digits before you begin entering digits in the *thousands* period.

Next we will deal with a number that contains both a whole number part and a decimal part.

Suppose we are asked to write “three hundred two thousand, twenty and two hundred one ten-thousandths” using digits. The diagram below represents the result.

Three hundred two thousand, twenty and two hundred one ten-thousandths.



Notice that the digits 3, 0, and 2, are in the *thousands* period. This represents “three hundred two thousand”. In the *ones* period are the digits 0, 2, and 0, which represent twenty.

To the right of the decimal point are the digits 0, 2, 0, and 1. Because the digit 1 is the rightmost digit and is located in the ten-thousandths place, the decimal part .0201 represents “two-hundred one ten-thousandths”. We can also say that 201 terminates in the ten-thousandths place.

Example 4: Write each of the following numbers using digits.

- a) Three hundred one.

301 Here we have 301 in the ones period.

- b) One thousand and fifty-four hundredths.

1,000.54 Here we have 1 in the thousands period, three 0's in the ones period, and to the right of the decimal point, 54 terminates in the hundredths place.

- c) Two thousand, thirteen and eighty-seven one-thousandths.

2,013.087 Here we have 2 in the thousands period, 13 in the ones period, and to the right of the decimal point, 87 terminates in the one-thousandths place.

- d) Six hundred ninety-three billion, nine thousand and six one-millionths.

693,000,009,000.000006 Here we have 693 in the billions period, three 0's in the millions period, 9 in the thousands period, three 0's in the ones period, and to the right of the decimal point, 6 is in the one-millionths place.

For Exercises 5 – 10, write each of the numbers using words.

- 5) 687.05
- 6) 1,000.001
- 7) 32,870,051.369
- 8) 50,000,090.0030
- 9) 304,000,000,000
- 10) 0.000050801

For Exercises 11 – 16, write the number using digits.

- 11) Three and five hundredths.
- 12) Sixteen ten-thousandths.
- 13) Four million and one one-thousandths.
- 14) Two hundred-thousandths.
- 15) Nine thousand and nine hundred hundred-thousandths.
- 16) Thirty-two thousand, eight hundred one-millionths.

Review Exercises

Evaluate the expression.

- 17) $9 - 5 + 4$
- 18) $4 - 8 + 2$
- 19) $3 - |-2 - 4|$
- 20) $|7 - 11| - 2^2$
- 21) -3^2
- 22) $(-3)^2$
- 23) $-(-5)^2$
- 24) $-|-6|^2$

For Exercises 25 – 28, find the value of each expression if $x = 3$ and $y = -2$.

- 25) $3 - x - y$
- 26) $x^2 + y^2$
- 27) $\frac{3 + y^3}{5 - x^2}$
- 28) $\frac{x}{2y} - \frac{2y}{x}$